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TIDES AND TIDAL CURRENTS IN THE STRAIT OF GIBRALTAR COMPUTED WI--ETC(U)
MAR 70 T LAEVASTU, W E HUBERT

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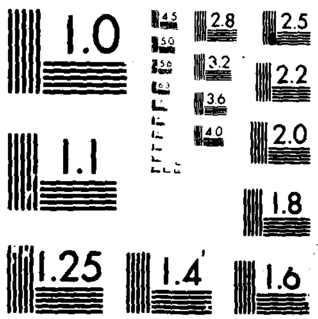

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(6) TIDES AND TIDAL CURRENTS IN THE STRAIT OF GIBRALTAR COMPUTED WITH THE HYDRODYNAMICAL NUMERICAL MODEL OF Walter HANSEN

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Abstract

The Hydrodynamical Numerical (HN) model of Walter Hansen has been used to compute tides and currents in the Strait of Gibraltar. The model has two open boundaries at which the tides were prescribed at each time step, using four tidal constituents. The grid size was two nautical miles, and the time step (from Courant criterion) was 24 seconds. Equilibrium was established after 20 hours of real-time computation. The main results from these model runs were: (1) At mean tides, the inflowing currents are stronger and last somewhat longer than the outflowing currents. (2) There is a net inflow into the Mediterranean Sea caused by tidal currents alone (excluding wind and sea level difference). (3) The tides computed with the HN model were in good agreement with those obtained from the harmonic method at points where harmonic constants were available. There was also a good agreement between computed currents and the few available current measurements. (4) The HN model gave a net transport of 1•48 km^3 cu km per hour into the Mediterranean in the upper 100 m. This value is somewhat higher than previous estimates; however, heat budget considerations require a higher water exchange through the Strait of Gibraltar than do previous estimates.

1. INTRODUCTION (Purpose of the paper).

The Hydrodynamical Numerical models of Professor Walter Hansen, University of Hamburg, have been tested on a number of shallow semi-closed seas. The model closely reproduces observed tides, storm surges and currents (Hansen, 1966; Sündermann, 1966; Laevastu and Stevens, 1969). It was desirable to test this model on the Strait of Gibraltar, which is a relatively deep area and has two open boundaries with distinctly different tides. It is of interest to know whether the latter aspect is properly reproduced by HN models. Furthermore, the Strait of Gibraltar is an ideal area for eventual testing of a two-layer HN model. This report presents the results and some verifications of single-layer HN model tests for the Strait of Gibraltar.

2. HYDRODYNAMICAL FORMULAS AND W. HANSEN'S FINITE DIFFERENCE METHOD.

The hydrodynamical numerical (HN) method for computation of currents and sea level changes was proposed in its present form by Hansen in 1956.

The following basic equations are used in the single-layer model:

$$\frac{\partial u}{\partial t} - fv - \nu \Delta u + \frac{r}{H} u \sqrt{u^2 + v^2} + g \frac{\partial \zeta}{\partial x} = X + \frac{\tau^{(x)}}{H} \qquad (1)$$

$$\frac{\partial v}{\partial t} + fu - \nu \Delta v + \frac{r}{H} v \sqrt{u^2 + v^2} + g \frac{\partial \zeta}{\partial y} = Y + \frac{\tau^{(y)}}{H} \qquad (2)$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} (Hu) + \frac{\partial}{\partial y} (Hv) = 0 \qquad (3)$$

$\tau^{(x)}$ (and $\tau^{(y)}$) are usually expressed as:

$$\tau^{(x)} = \lambda W_x \sqrt{W_x^2 + W_y^2} \qquad (4)$$

The bottom stress (friction) term in formulas 1 and 2 is:

$$\tau^{(b)} = \frac{r}{H} u \sqrt{u^2 + v^2} \; ; \; \frac{r}{H} v \sqrt{u^2 + v^2} \qquad (5)$$

The following symbols were used in the formulas above:

x, y	space coordinates
t	time
u, v	components of velocity
H	total depth ($H = h + \zeta$)
ζ	surface elevation
X, Y	components of external forces
$\tau^{(x)}, \tau^{(y)}$	components of wind stress ($\lambda = 3.5 \times 10^{-6}$)
g	acceleration of gravity
f	Coriolis parameter
r	friction coefficient (3×10^{3}) (bottom stress)
ν	coefficient of horizontal eddy viscosity
∇	Laplace operator
λ	coefficient of friction (drag coefficient)
W_x, W_y	wind speed components
$\tau^{(b)}$	bottom stress

Analytical solution(s) to formulas 1 to 3 are of little value, as exact solutions are possible only for basins of regular shape, simple

depth and simple wind distribution. However, the formulas can be solved on computers using "step-by-step" finite-difference methods.

$$\zeta^{t+\tau}(n,m) = \bar{\zeta}^{t-\tau}(n,m) - \frac{\tau}{\ell}\Big\{H_u^t(n,m)\,U^t(n,m) - H_u^t(n,m-1)\,U^t(n,m-1) + H_v^t(n-1,m)\,V^t(n-1,m) - H_v^t(n,m)\,V^t(n,m)\Big\} \tag{6}$$

$$U^{t+2\tau}(n,m) = \Big\{1 - \big[2\tau r\ /H_u^{t+2\tau}(n,m)\big]\sqrt{\bar{U}^t(n,m)^2 + V^{*t}(n,m)}\Big\}\ \bar{U}^t(n,m) + 2\tau f V^{*t}(n,m) - \frac{\tau g}{\ell}\Big\{\zeta^{t+\tau}(n,m+1) - \zeta^{t+\tau}(n,m)\Big\} + 2\tau X^{t+2\tau}(n,m) \tag{7}$$

$$V^{t-2\tau}(n,m) = \Big\{1 - \big[2\tau r/H_v^{t-2\tau}(n,m)\big]\sqrt{\bar{V}^t(n,m)^2 + U^{*t}(n,m)^2}\Big\}\,\bar{V}^t(n,m) - 2\tau f U^{*t}(n,m) - \frac{\tau g}{\ell}\Big\{\zeta^{t+\tau}(n,m) - \zeta^{t+\tau}(n+1,m)\Big\} + 2\tau Y^{t+2\tau}(n,m) \tag{8}$$

The "averaged" velocity and water elevation (sea level) components are:

$$\bar{U}^t(n,m) = \alpha U^t(n,m) + \frac{1-\alpha}{4}\Big\{U^t(n-1,m) + U^t(n+1,m) + U^t(n,m+1) + U^t(n,m-1)\Big\}$$

$\bar{V}^t(n,m)$ and $\bar{\zeta}^t(n,m)$ are analogous. (9)

(The factor can be interpreted as "horizontal viscosity parameter." Its normal value is 0.99 (see further Chapter 4.1))

$$U^{*t}(n,m) = \frac{1}{4}\Big\{U^t(n,m-1) + U^t(n+1,m-1) + U^t(n,m) + U^t(n+1,m)\Big\} \tag{10}$$

$V^{*t}(n,m)$ is analogous to the $U^{*t}(n,m)$ above.

The time step is 2τ. The total depth (H_u, H_v) is computed as

$$H_u^{t+2\tau}(n,m) = h_u(n,m) + \frac{1}{2}\left\{\zeta^{t+\tau}(n,m) + \zeta^{t+\tau}(n,m+1)\right\} \qquad (11)$$

The effects of wind (external force) are computed with the following formula:

$$X^t = \frac{\lambda W_x^t \sqrt{(W_x^t)^2 + (W_y^t)^2}}{H} - \frac{1}{\rho r} \frac{\partial p_o}{\partial x} \qquad (12)$$

The effects of wind (external force) are computed with the following formula:

$$Y^t = \frac{\lambda W_x^t \sqrt{(W_x^t)^2 + (W_y^t)^2}}{H} - \frac{1}{\rho r} \frac{\partial p_o}{\partial x} \qquad (13)$$

A number of slightly modified finite difference schemes are possible for solving the hydrodynamical equations. Some of these schemes are being tested at FNWC.

3. HYDRODYNAMICAL NUMERICAL (HN) MODEL OF STRAIT OF GIBRALTAR.

The computational grid for the Strait of Gibraltar has a two nautical mile mesh length. This grid size and the maximum depth in the area require (according to Courant, Friedrich, Lewy criterion) a time step of 24 seconds in order that the computations remain stable. Depths at u and v grid points were obtained from navigation charts. The friction coefficient was 0.03. Tides with four tidal constituents (M_2, S_2, N_2 and K_2) were introduced at each time step at both ends. The tidal input was constant across the openings. It has been found at FNWC that the tidal input at the openings can be made an inverse function of depth. However, if this is not done, the program will adjust the z and u, v values a few gridpoints inwards. Equilibrium in this particular program was established in about 20 hours of real time. The program was run 100 hours. Mean tides (tides between spring and neap) and the hourly outputs were taken from the last 25 hours of computation. The program was also run with wind inputs, but essentially calm wind results are reported in this paper.

4. VERIFICATION OF SEA LEVEL COMPUTATIONS.

The tides at a number of locations for which tidal harmonic constants were available were computed with the well-known harmonic method. The same time span as the output from the HN method was used as well as the same four tidal constituents. The sea level changes computed with the HN method were taken from a grid point closest to the original location of the corresponding tide gauge

constituents ~~and~~ used in the harmonic computations. Some comparisons are shown in Figures 1 to 4.

At Tangier (Figure 1), there is virtually no time lag between the HN and harmonic models. However, the HN model gives about 10% higher amplitude, the difference occurring mainly in low water prediction. At Tarifa (Figure 2), the tides with the HN method precede about 10 minutes the tides from the harmonic method and the HN tides are about 8% higher. At Gibraltar (Figure 3), and at Ceuta (Figure 4), the tidal heights of both methods are in relatively good agreement; however, the HN tides precede the "harmonic" tides by about 30 minutes. It should be noted that the HN tides are taken from grid points which are as much as two miles away from the tide gauges and usually over deeper water.

5. VERIFICATION OF COMPUTED CURRENTS.

In addition to producing tidal forecasts in areas where harmonic constants are not available, the HN model has the advantages of handling superimposed meteorological tides (storm surges) and of predicting tidal currents. The latter were not obtainable using "harmonic" methods unless current recordings were available over relatively long intervals.

Examples of synoptic tidal currents in the Strait of Gibraltar are given in Figures 5 and 6. The direct verification of synoptic currents is difficult due to the scarcity of current measurements. Some quasi-synoptic current charts have been prepared subjectively using a variety of data, including ships' logbooks. Gaps have often been filled in using some degree of "artistic license." Two subjective tidal current charts for the Strait of Gibraltar are given in Figures 7

and 8 (from NOO Publ. 700), corresponding to the times and currents in Figures 5 and 6. Although detailed comparison of the two sets of charts has no direct use, it can be noted that agreement is good.

A more detailed comparison of measured and computed tidal currents at a given location in the western entrance to the Strait of Gibraltar is given in Figure 9. Although the exact stage of tides (mean, spring or neap) during the measurement is not known, the relation between the observed and computed tides is a good one. The tidal currents, computed with the HN method are about 20% stronger.

It should be noticed that the tidal currents are considerably weaker in the Bay of Cadiz than at the entrance to and inside the Strait of Gibraltar. An example of currents in the Bay of Cadiz computed with another HN model is given in Figure 10.

6. TRANSPORT OF WATER THROUGH THE STRAIT OF GIBRALTAR.

The general circulation in the Strait of Gibraltar (inflow of Atlantic water at the surface, outflow of Mediterranean water along the bottom) has been discussed in a number of publications and various quantitative estimates of the water exchange have been given (e.g. Wüst 1952, 1959; Lacombe 1961, and others). The possible influence of different forces in effecting the seasonal changes of the water exchange, such as atmospheric pressure difference, winds, difference in evaporation in the Mediterranean, etc., have also been discussed. However, as far as can be ascertained by the authors, the net (or rest) tidal circulation in the water exchange in the Strait of Gibraltar has not been pointed out earlier.

Due to the variation of amplitude and phase of the tide over some distance, there will be a net transport of tidal currents. This net circulation over a mean tidal cycle (24h 50 min) in the Strait of Gibraltar is shown in Figure 11. As seen from this figure, the tides alone might effect the net inflow to the Mediterranean. This is due to the amplitude difference at opposite ends of the straits.

The best measurements of currents in the Strait of Gibraltar have been made by Lacombe (1961). The net currents, computed from these measurements, are shown in Figures 12 and 13. As seen, the main inflow occurs above 100m depth.

The transport of water through the straits from 0 to 100m during a tidal cycle was extracted from the HN model and is presented in Figure 14. The net inflow is 1.43 km^3/hour. This value is about half of that obtained by Lacombe from direct measurements (2.7 km^3 per hour), but about ten times as high as Lacombe's adjusted value (0.15 km^3 per hour).

If the net evaporation per square centimeter in the Mediterranean is about 150 cm/year, and all the water deficit were to be balanced by the inflow through the Strait of Gibraltar, a maximum net inflow of 0.45 km^3/hour would be required. Furthermore, heat budget computations at FNWC for the Mediterranean require a larger water exchange than heretofore estimated. More accurate insight into these problems is expected from the two-layer HN model.

7. SUMMARY.

a. The HN model with two open boundaries can be used for computation of tides and currents (including wind currents) in relatively deep straits.

b. The tidal phases and amplitudes computed with the HN model for the Strait of Gibraltar verify well. The maximum difference has been 30 minutes in time of high water and 20% in amplitude.

c. The net (or rest) tidal current gives a net inflow into the Mediterranean. The inflowing tidal currents are stronger and also last somewhat longer than the outflowing tidal currents.

d. The computed net inflow is somewhat larger than previously available estimates. However, heat budget computations require a large water exchange. Multilayer HN models will shed further light on this problem.

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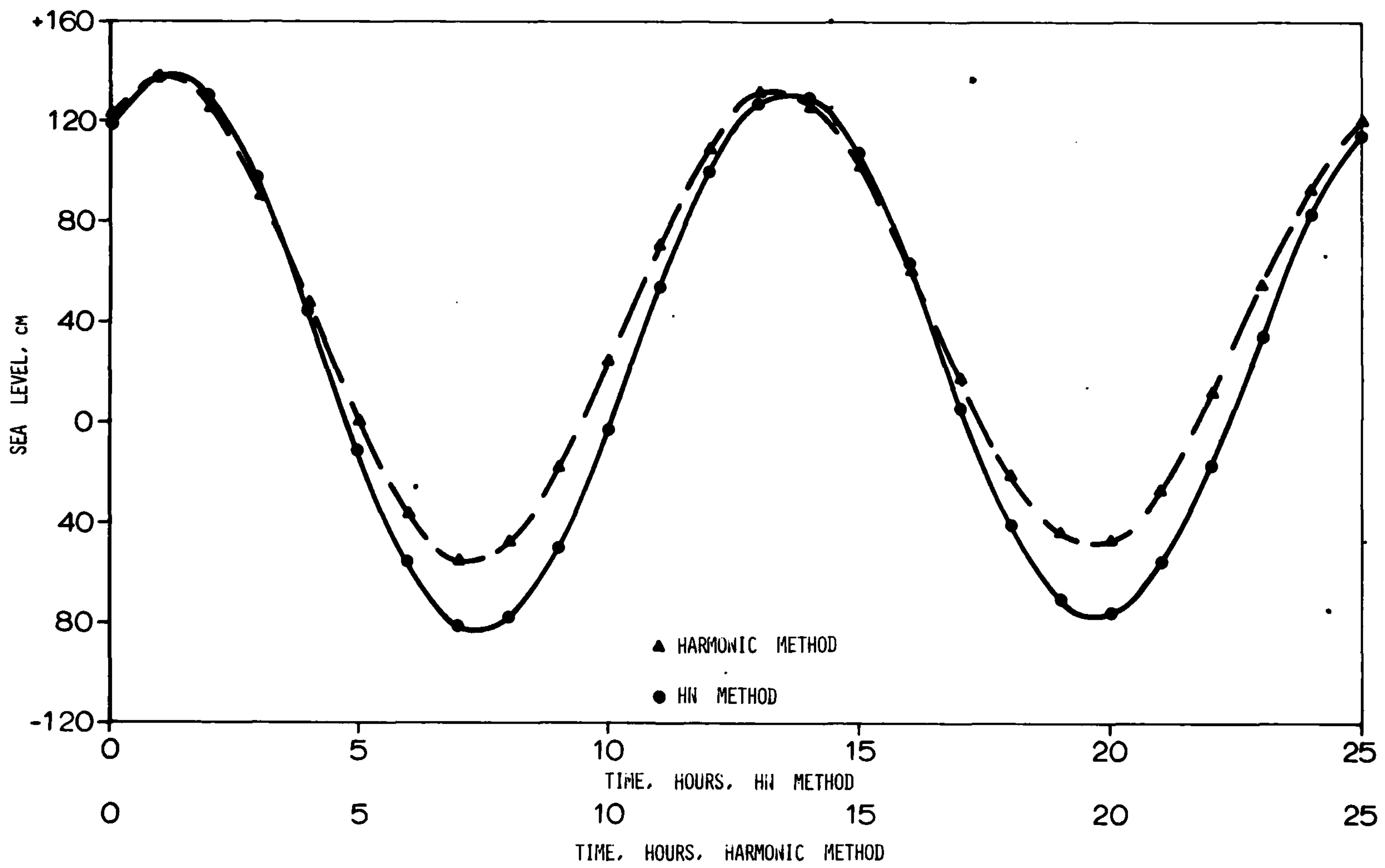


FIGURE 1. TIDES AT TANGIER, COMPUTED WITH HN AND HARMONIC METHODS (M2, S2, N2, K2).

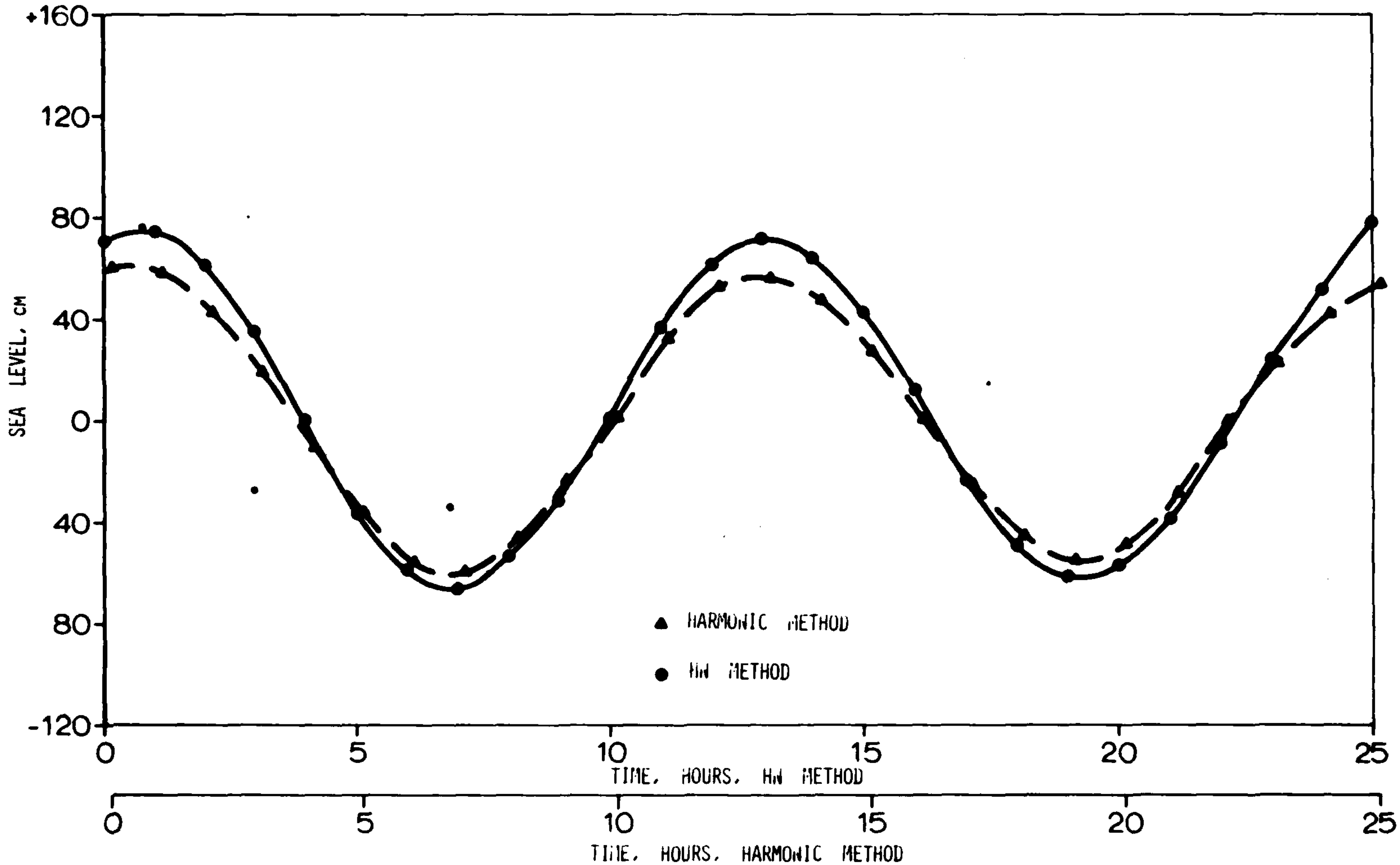


FIGURE 2. TIDES AT TARIFA, COMPUTED WITH HW AND HARMONIC METHODS (M2, S2, N2, K2).

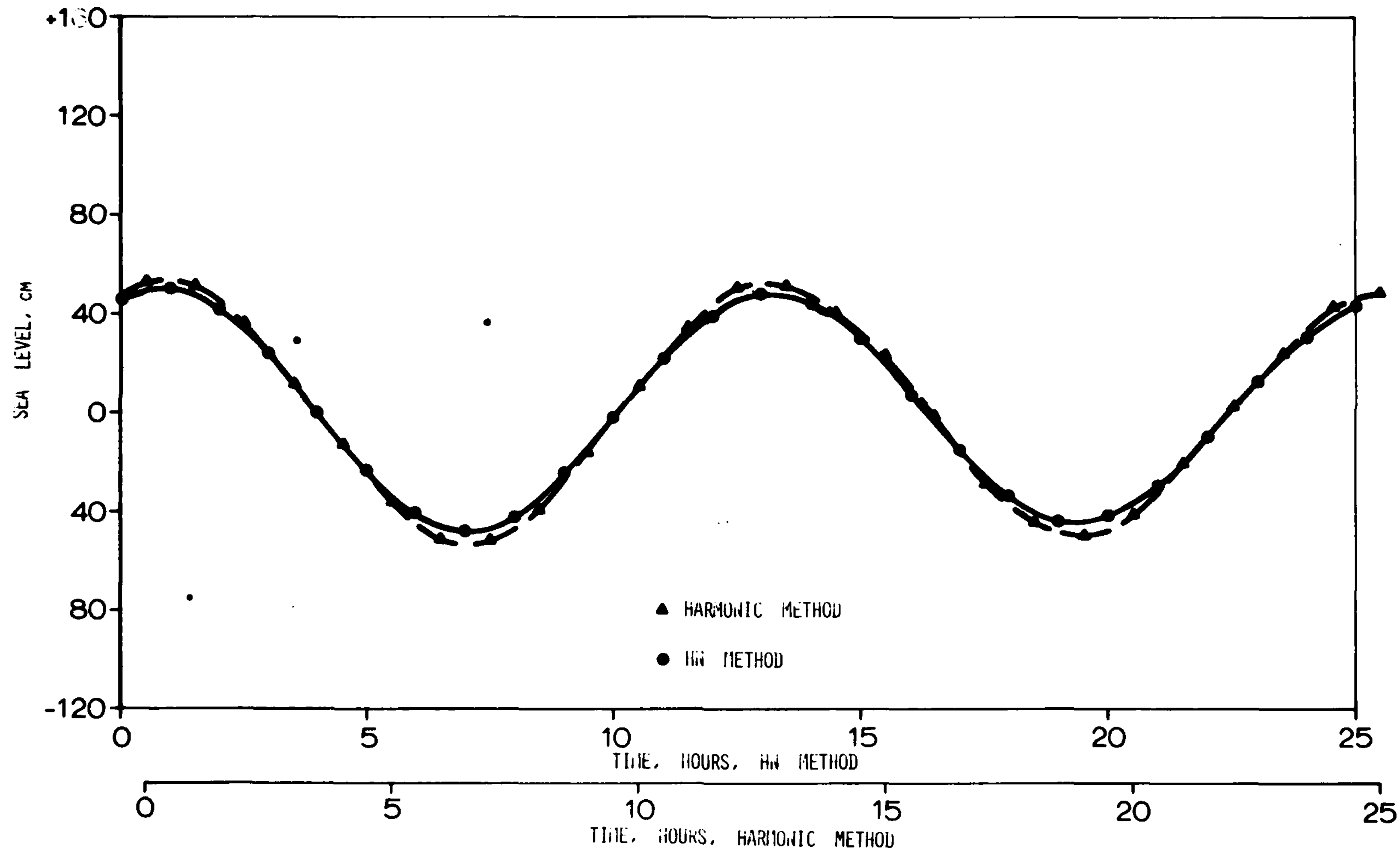


FIGURE 3. TIDES AT GIBRALTAR, COMPUTED WITH HN AND HARMONIC METHODS (M2, S2, N2, K2).

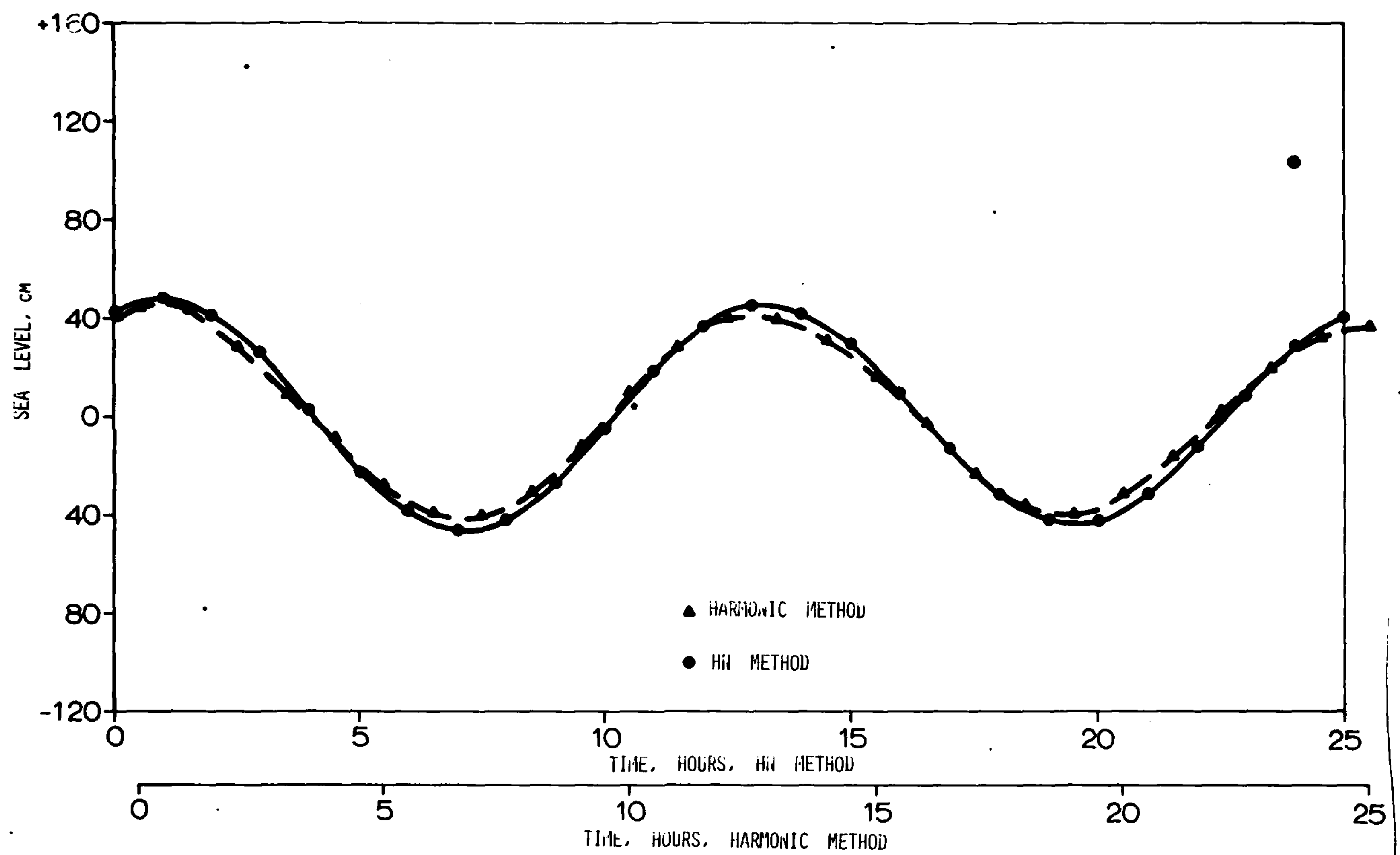


FIGURE 4. TIDES AT CEUTA, COMPUTED WITH HN AND HARMONIC METHODS (M2, S2, N2, K2).

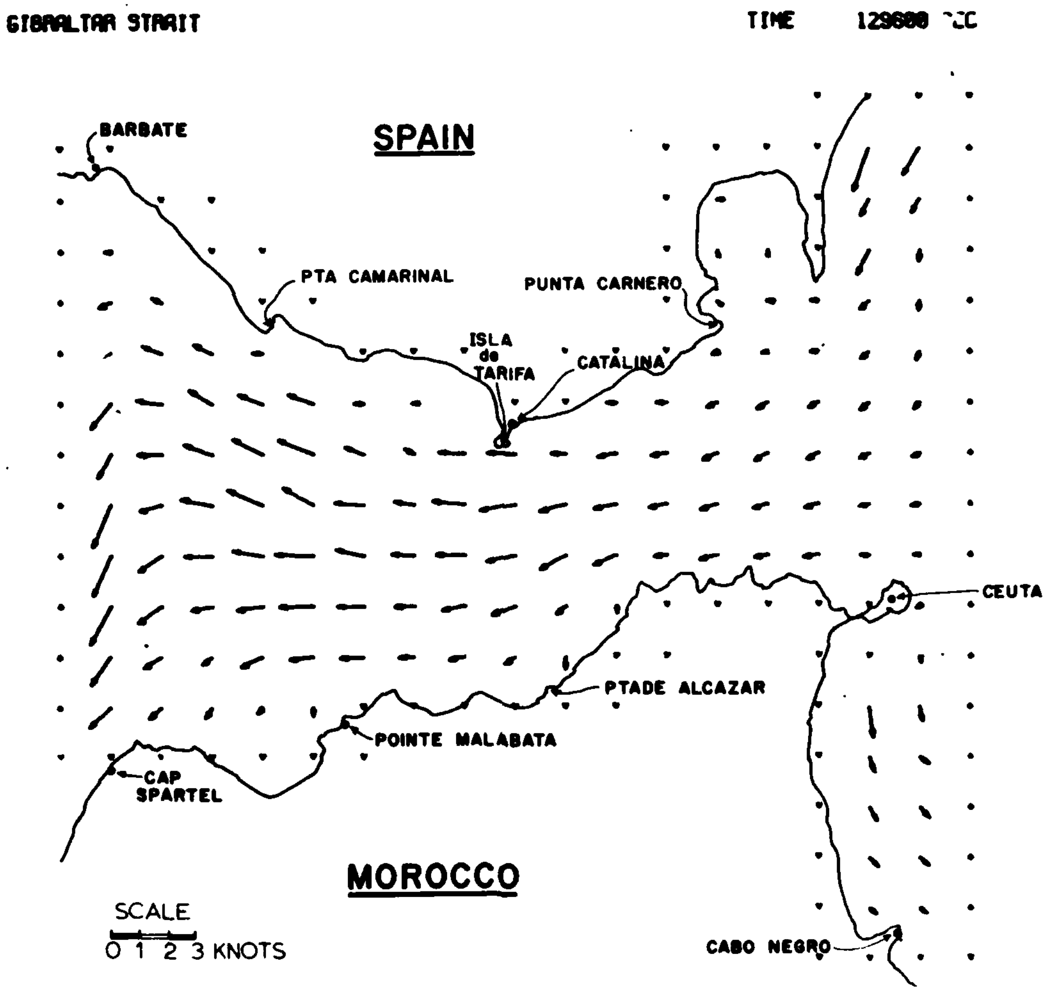


Figure 5 Tidal currents in Strait of Gibraltar three hours after low water at Tarifa. (HN method).

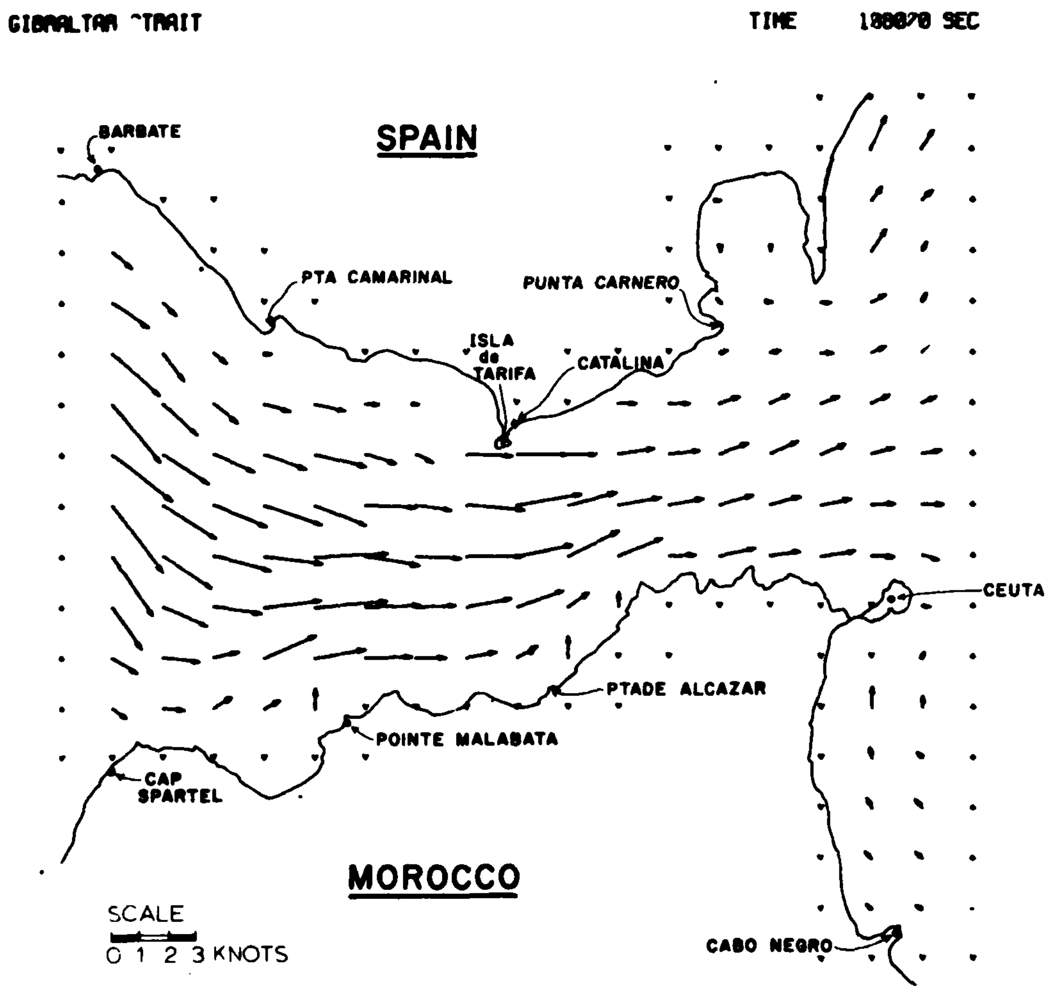


Figure 6 Tidal currents in Strait of Gibraltar three hours after high water at Tarifa. (HN method).

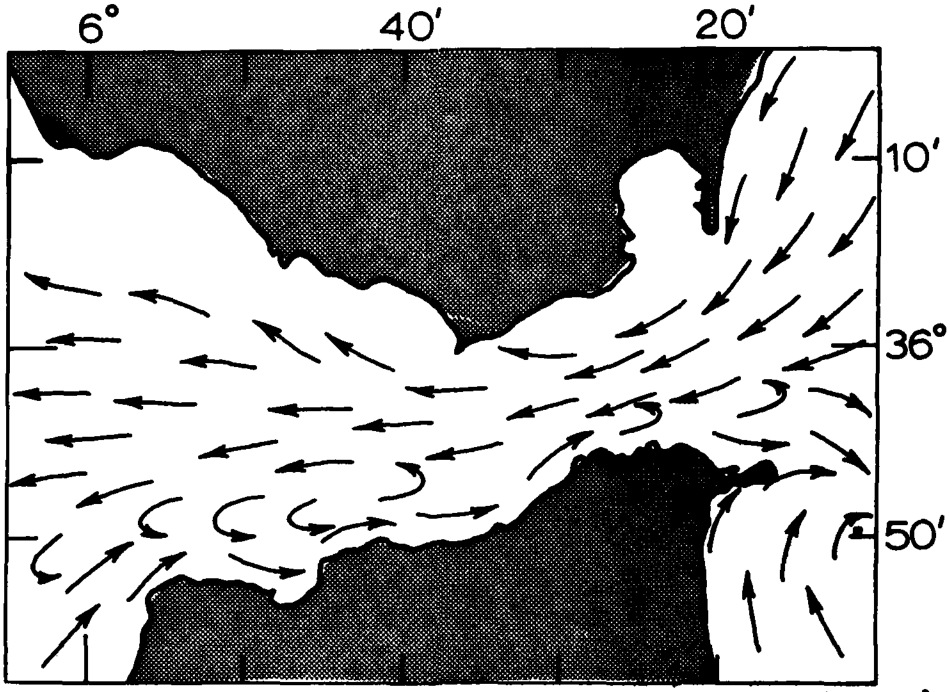


FIGURE 7. TIDAL CURRENTS IN STRAIT OF GIBRALTAR THREE HOURS BEFORE HIGH WATER AT GIBRALTAR (SUBJECTIVE SUMMARY, NOO PUBL. 700).

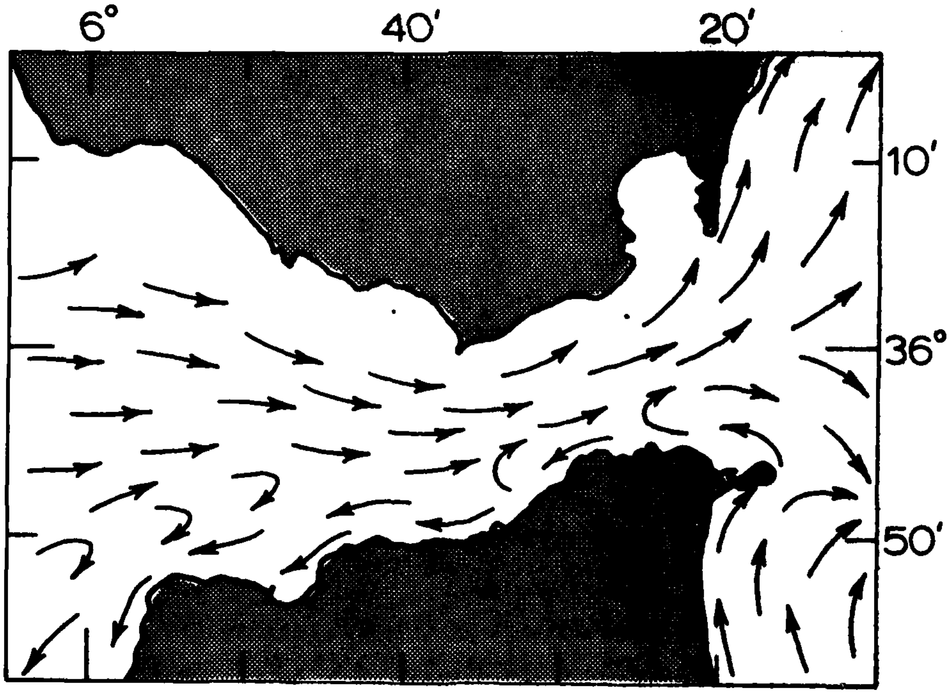


FIGURE 8. TIDAL CURRENTS IN STRAIT OF GIBRALTAR THREE HOURS AFTER HIGH WATER AT GIBRALTAR (SUBJECTIVE SUMMARY, NOO PUBL. 700).

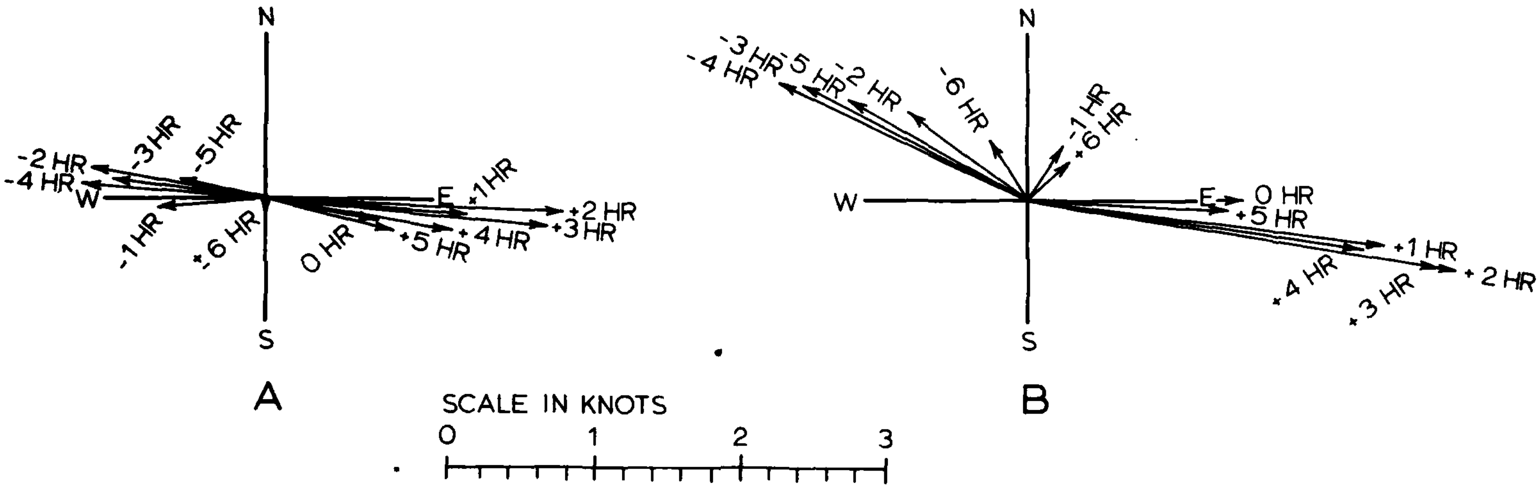


FIGURE 9. A. MEASURED TIDAL CURRENTS AT 250M AT 35 54 N, 5 52 W (NOO PUBL. 700). B. CURRENTS COMPUTED WITH HN METHOD AT ABOUT THE SAME LOCATION. TIMES INDICATE HOURS BEFORE AND AFTER HIGH WATER AT GIBRALTAR.

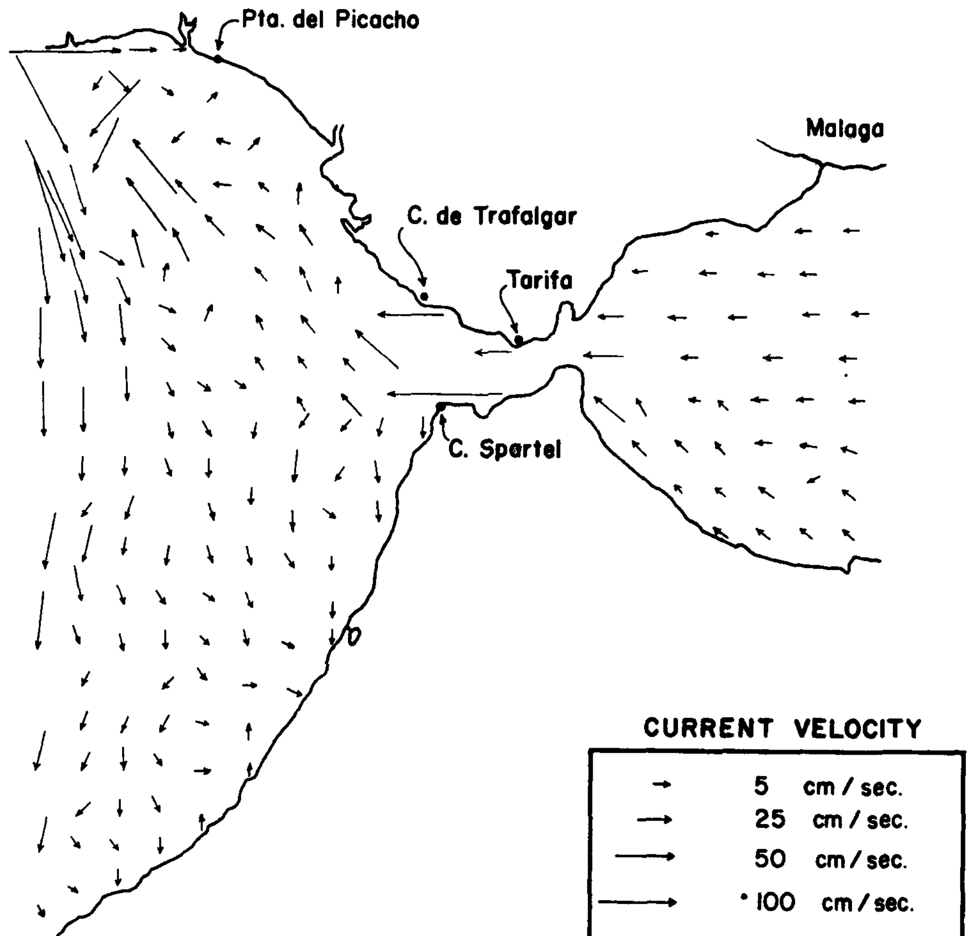


Figure 10 Currents (tidal and wind) in Bay of Gadiz at low water at Tarifa, computed with HN model (wind 6m sec^{-1} from E).

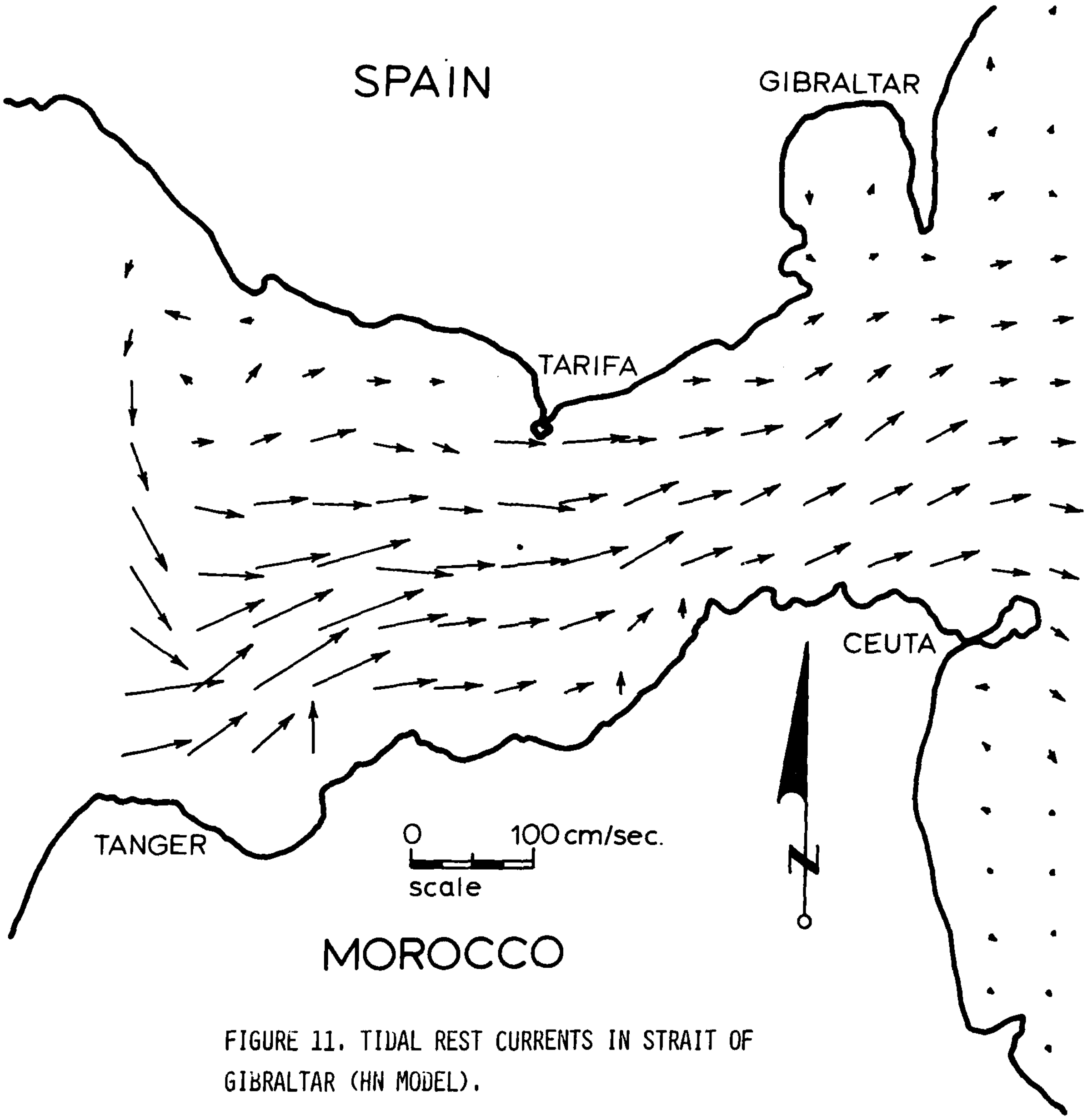
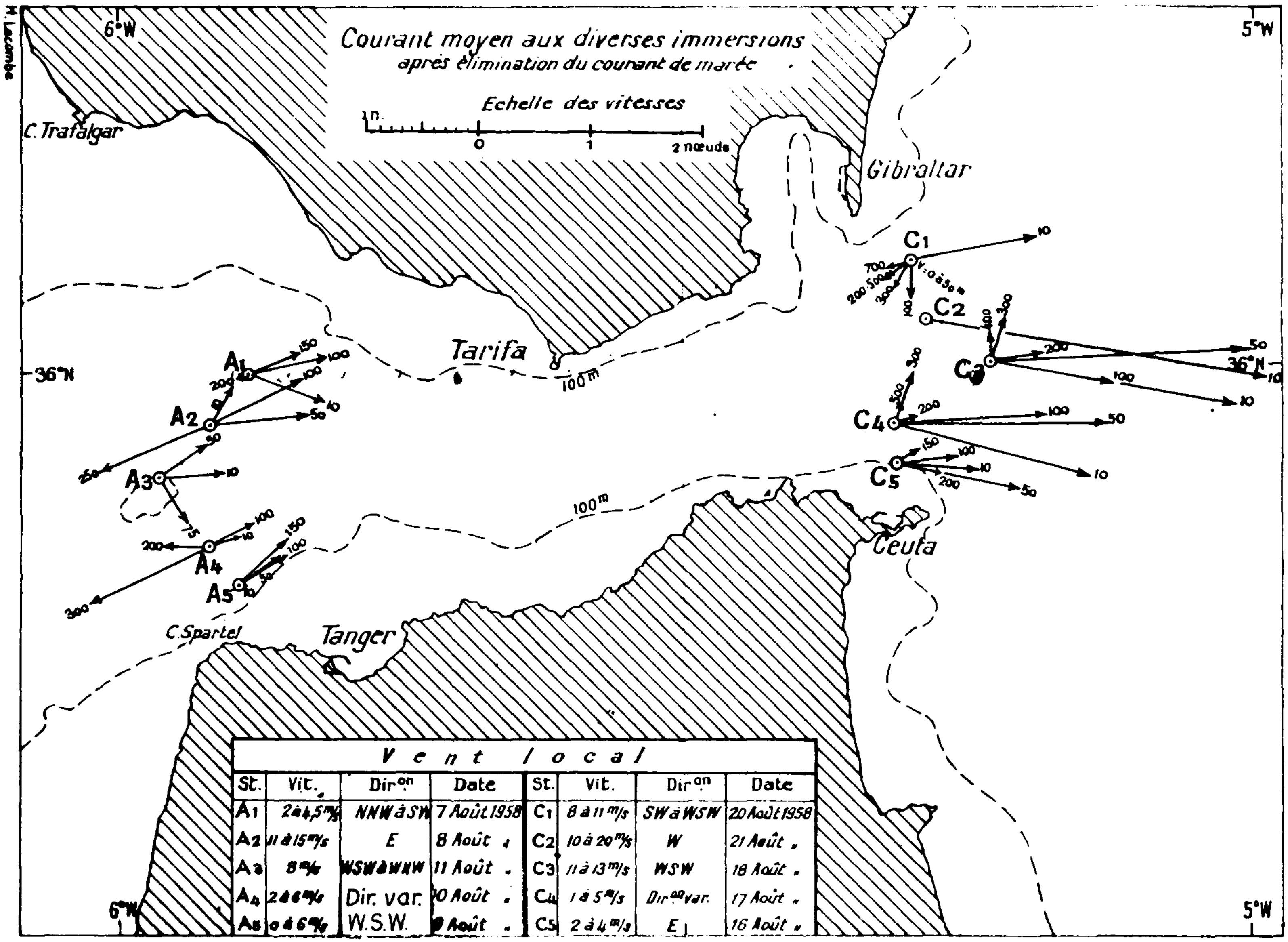


FIGURE 11. TIDAL REST CURRENTS IN STRAIT OF GIBRALTAR (HN MODEL).



Vent local							
St.	Vit.	Dir^on	Date	St.	Vit.	Dir^on	Date
A1	2 à 4,5 m/s	NNW à SW	7 Août 1958	C1	8 à 11 m/s	SW à WSW	20 Août 1958
A2	11 à 15 m/s	E	8 Août "	C2	10 à 20 m/s	W	21 Août "
A3	8 m/s	WSW à WNW	11 Août "	C3	11 à 13 m/s	WSW	18 Août "
A4	2 à 6 m/s	Dir. var.	10 Août "	C4	1 à 5 m/s	Dir^on var.	17 Août "
A5	0 à 6 m/s	W.S.W.	9 Août "	C5	2 à 4 m/s	E	16 Août "

FIGURE 12. MEASURED REST CURRENTS IN STRAIT OF GIBRALTAR (AFTER LACOMBE 1961).

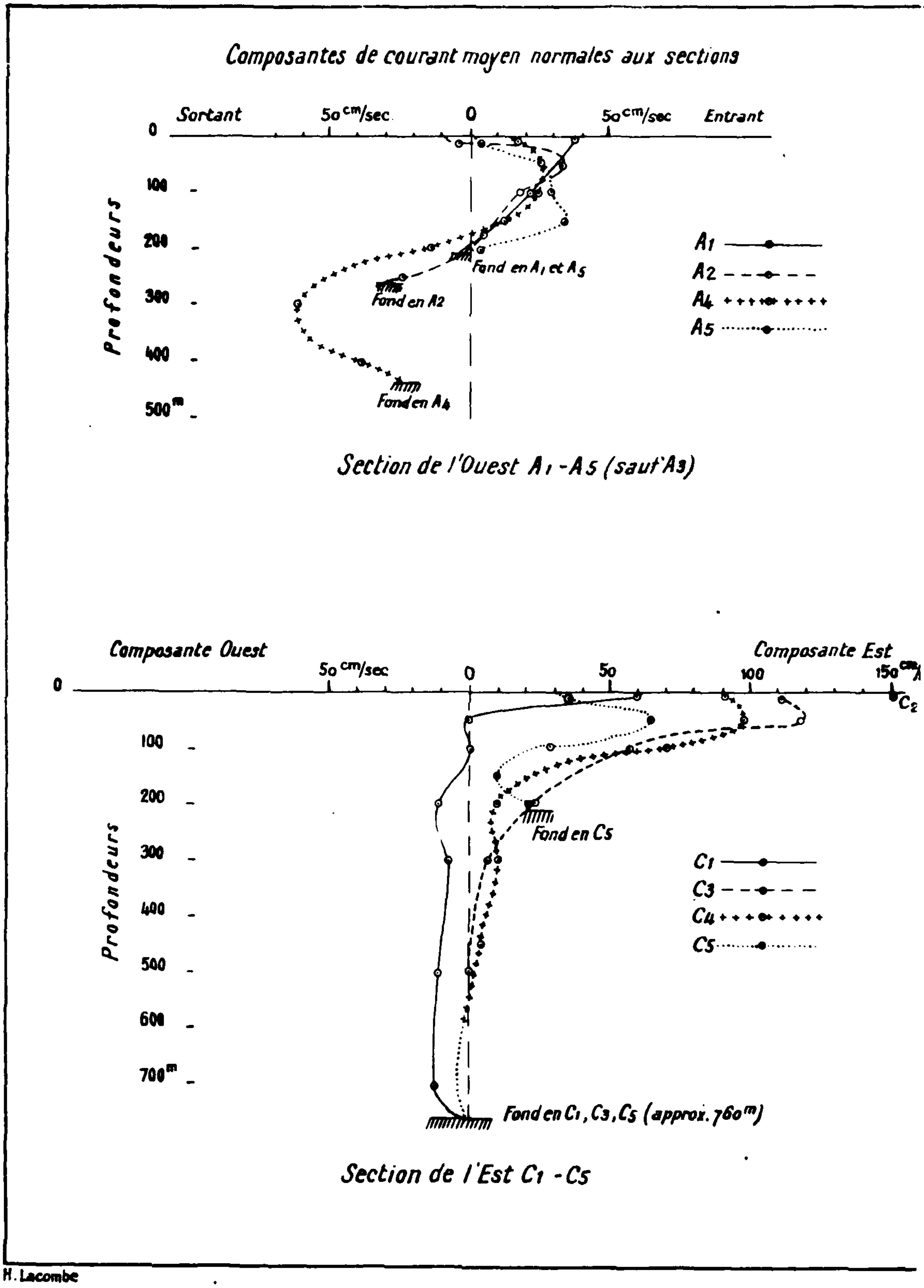


FIGURE 13. MEASURED REST CURRENTS AT DIFFERENT DEPTH IN STRAIT OF GIBRALTAR (LOCATIONS SEE FIG. 12, AFTER LACOMBE 1961).

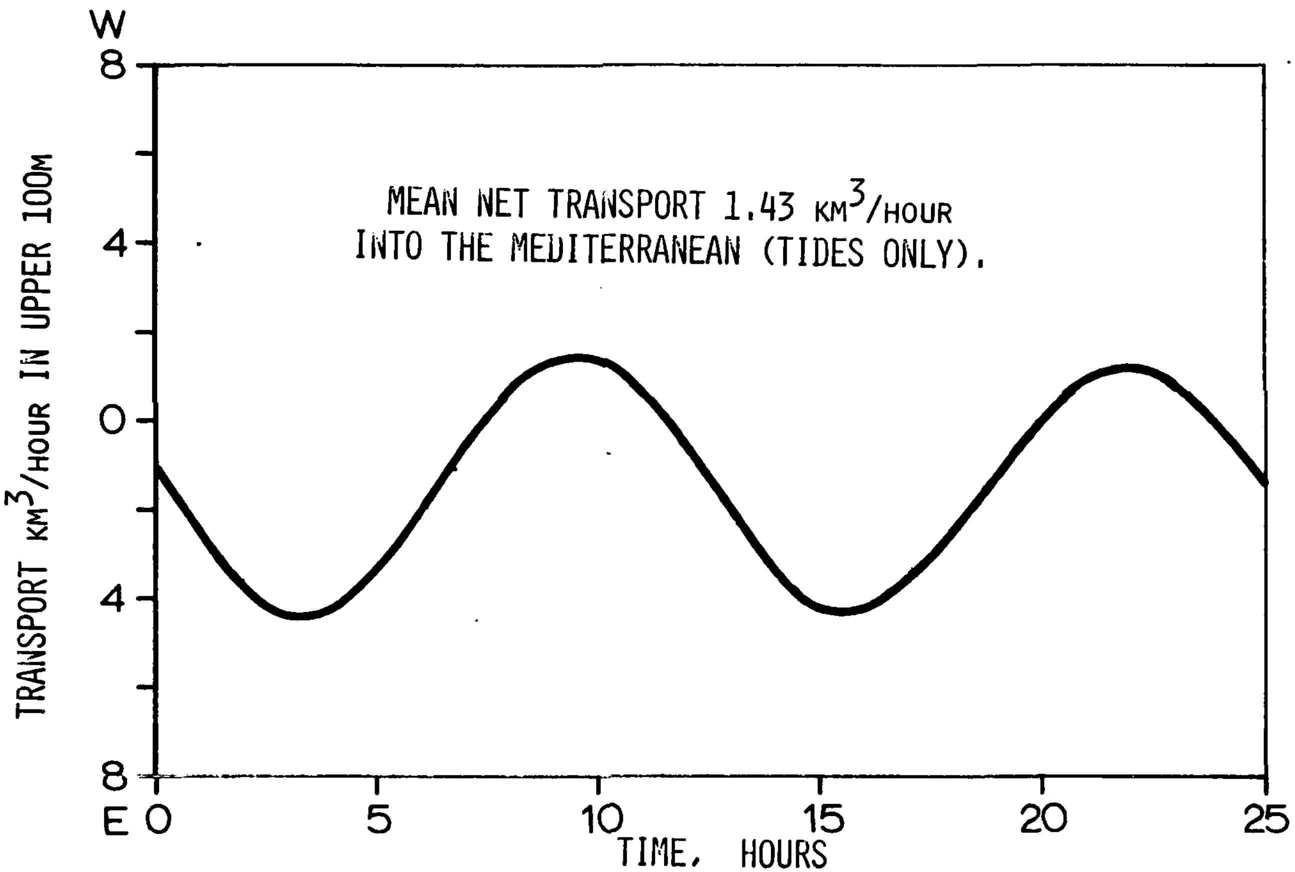


FIGURE 14. TRANSPORT OF WATER THROUGH STRAIT OF GIBRALTAR FROM SURFACE TO 100M. (HN MODEL)

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